Department of Mathematics, IIT Madras

MA-5895-Numerical Optimization

Assignment 2

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Q. 1 Implement the conjugate gradient (refined version) algorithm and use to it solve linear systems in which A is the Hilbert matrix, whose elements are

$$A_{i,j} = \frac{1}{i+j-1}.$$

Set the right-hand-side to $b^T = (1, 1, \ldots, 1)$, and the initial point to $x_0 = 0$. Try dimensions $n = 5, 8, 12, 20$ and report the number of iterations required to reduce the residual below 10^{-7}.

Q. 2 Consider the problem:

$$\min_{x \in \mathbb{R}^2} f(x) \;=\; -2x_1 + x_2 \quad \text{subject to} \begin{cases} (1-x_1)^3 - x_2 & \geq 0 \\ x_2 + \frac{1}{4}x_1^2 - 1 & \geq 0 \end{cases} \tag{1}$$

The optimal solution is $x^* = (0, 1)^T$, where both constraints are active.

(a) Verify if the LICQ hold at this point.

(b) Verify if the KKT conditions satisfied.

(c) Write down the sets $\mathcal{F}(x^*)$ and $\mathcal{C}(x^*, \lambda^*)$.

(d) Are the second-order necessary conditions satisfied? Are the second-order sufficient conditions satisfied?

Q. 3 Show that the dual of the dual of a linear program is the primal problem.

Q. 4 Consider the following linear program:

$$\begin{aligned} \min -5x_1 - x_2 \quad &\text{subject to} \\ x_1 + x_2 &\leq 5, \\ 2x_1 + \frac{1}{2}x_2 &\leq 8, \\ x &\geq 0. \end{aligned} \tag{2}$$

(a) Add slack variables x_3 and x_4 to convert this problem to standard form.

(b) Solve this problem using the simplex method, showing at each step the basis and the vectors λ, s_N and x_B, and the value of the objective function.

Hint: The initial choice of $\mathcal{B}$ for which $x_B \geq 0$ should be obvious once you have added the slacks in part (a).