Department of Mathematics, IIT Madras

MA-5895-Numerical Optimization

Problem Sheet 1

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Q. 1 Suppose that $f(x) = x^T Qx$, where $Q \in \mathbb{R}^{n\times n}$ is a symmetric positive semi-definite matrix and $x \in \mathbb{R}^n$. Show that f is convex on the domain $\mathbb{R}^n$.

Q. 2 Define a line search algorithm. What are the key components for a line search method. What are the necessary conditions on these components, violating those the algorithm fails to converge.

Q. 3 Suppose that $\tilde{f}(z) = f(x)$ where $x = Sz + s$ for some $S \in \mathbb{R}^{n\times n}$ and $s \in \mathbb{R}^n$. Show that

$$\nabla \tilde{f}(z) = S^T \nabla f(x), \qquad \nabla^2 \tilde{f}(z) = S^T \nabla^2 f(x) S.$$

Q. 4 Let $f : \mathbb{R}^n \to \mathbb{R}$. Assume that f is twice continuously differentiable. Explicitly derive the steepest descent direction i.e. the direction in which maximum reduction takes place from any point x_0.

Q. 5 Write down the conditions to be enforced on the matrix B which is an approximation to the Hessian in a quasi Newton method, for the search direction to be a descent direction. Show that under these conditions the quasi Newton direction is a descent direction.

Q. 6 Suppose f is the following quadratic function

$$f(x) = \frac{1}{2} x^T Qx - b^T x,$$

where Q is symmetric and positive definite. Find the minimiser of the function. Prove that it is unique. Compute α such that it uniquely minimises the univariate $\phi(\alpha) = f(x - \alpha\nabla f)$ for any fixed x.

Q. 7 Let the cost function of the unconstrained optimization problem of interest be

$$f(x) = 2x_1^2 + x_1x_2 + x_2^2 + x_2x_3 + x_3^2 - 6x_1 - 7x_2 - 8x_3 + 9$$

Recall that the steepest descent algorithm is

$$x_{k+1} = x_k - \alpha_k \nabla f(x_k)$$

$\alpha_k \in \mathbb{R}_{>0}$.

(a) Using $x_0 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, write out the first iteration of the steepest descent algorithm and obtain the optimum value of α_0.

(b) What is the value of x_1 if you implement α_0?

(c) Verify that $f(x_1) < f(x_0)$?