Department of Mathematics, IIT Madras

MA-5895-Numerical Optimization

Problem Sheet 4

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Q. 1 Consider the problem

$$\min_{(x_1,x_2)} \frac{10x_2^2 - x_1^2 + 8x_1 - 16}{10} \quad \text{subject to} \quad x_1^2 + x_2^2 \geq 1. \tag{1}$$

Show that the objective function is unbounded below, globally. As a consequence search for a local solution with respect to the prescribed constraint by:

(a) first finding a candidate x^* which satsify the first order necessary condtions (KKT);

(b) inturn show that this candidate satifies the second order sufficient condition aswell.

Q. 2 Consider the problem

$$\min_{x} \left(x_1 - \frac{3}{2}\right)^2 + (x_2 - t)^4 \quad \text{s.t.} \begin{bmatrix} 1 - x_1 - x_2 \\ 1 - x_1 + x_2 \\ 1 + x_1 - x_2 \\ 1 + x_1 + x_2 \end{bmatrix} \geq 0. \tag{2}$$

(a) Find value(s) of t for which the point $x^* = (1, 0)^T$ satisfy the KKT conditions.

(b) Show that when $t = 1$, only the first constraint is active at the solution, and find the solution.

Q. 3 Consider the linear program:

$$\begin{aligned} \min_{x} \quad & 2x_1 + x_3 + x_4 \\ \text{subject to} \quad & x_1 + x_2 + x_3 = 1 \\ & x_2 + x_3 + x_4 = 2 \\ & x_1 \geq 0 \\ & x_2 \geq 0 \\ & x_3 \geq 0 \\ & x_4 \geq 0 \end{aligned} \tag{3}$$

(a) Write down the dual problem for the LP above.

(b) Express the dual problem in part (a) as a standard LP of the form:

$$\begin{aligned} \min_{\pi} \quad & c^T \pi \\ \text{subject to} \quad & A\pi = b \\ & \pi \geq 0. \end{aligned} \tag{4}$$

Comment about the relation between the maximiser of the dual problem and the solution of the problem (4).

Q. 4 Convert the following linear program to standard form:

$$\max_{x,y} c^T x + d^T y \ \text{ subject to } A_1 x = b_1,\ A_2 x + B_2 y \leq b_2,\ l \leq y \leq u, \tag{5}$$

where there are no explicit bounds on the optimisation vector.

Q. 5 Show that the dual of the linear program

$$\min c^T x \quad \text{subject to } \ Ax \geq b,\ x \geq 0, \tag{6}$$

is

$$\max b^T \lambda \quad \text{subject to } \ A^T \lambda \leq c, \lambda \geq 0. \tag{7}$$

Q. 6 Show the equivalence of the KKT conditions for the primal (6) and dual (7) problems.