

Department of Mathematics, IIT Madras

MA-5895-Numerical Optimization

Problem Sheet 5

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Q. 1 Consider the following optimisation problem:

$$\min_x f(x) \quad \text{subject to } Ax = b, \quad (1)$$

where A is a $m \times n$ matrix and $m \leq n$ and x is a vector in \mathbb{R}^n and b is a vector in \mathbb{R}^m . If A has full row rank show that (1) is equivalent to an unconstrained optimisation problem given by:

$$\min_{x_N} h(x_N) \stackrel{\text{def}}{=} f \left(P \begin{bmatrix} B^{-1}b - B^{-1}Nx_N \\ x_N \end{bmatrix} \right), \quad (2)$$

where $x_B \in \mathbb{R}^m$ and $x_N \in \mathbb{R}^{n-m}$ are some appropriate vectors, P , B and N are respectively $n \times n$, $m \times m$ and $(n-m) \times m$ matrices.

Q. 2 Consider the quadratic program:

$$\begin{aligned} \min_x q(x) &= \frac{1}{2}x^T Gx + x^T c \\ \text{subject to } Ax &\geq b, \quad \bar{A}x = \bar{b}, \end{aligned} \quad (3)$$

where G is symmetric and positive semidefinite. Write down the KKT conditions for the above problem.

Q. 3 Consider the following quadratic optimisation problem

$$\min_x q(x) = (x_1 - 1)^2 + (x_2 - 2.5)^2 \quad (4)$$

$$\text{subject to } x_1 - 2x_2 + 2 \geq 0, \quad (5)$$

$$-x_1 - 2x_2 + 6 \geq 0, \quad (6)$$

$$-x_1 + 2x_2 + 2 \geq 0, \quad (7)$$

$$x_1 \geq 0, \quad (8)$$

$$x_2 \geq 0. \quad (9)$$

Refer the constraints, in order, by indices 1 through 5. Starting with the feasible point $x^0 = (2, 0)^T$ and the initial working set $\mathcal{W}_0 = \{3\}$. Write iterations of primal active-set method and return solve the problem under consideration.